1901. Multiply the two brackets together first, leaving the squares until afterwards. When squaring, use $(3^x)^2 \equiv 3^{2x} \equiv (3^2)^x \equiv 9^x$.

1902. Find "the best linear approximation" is equivalent to finding the tangent line $y = mx + c$.

1903. Substitute the two parametric equations into the equation of the circle, and solve. Use the double-angle formula $\sin 2t \equiv 2 \sin t \cos t$ to produce a quadratic in $\sin^2 t$. This factorises.

1904. Remember that a histogram (like almost all stats) is trying to represent the underlying *population*, not the particular sample you happen to have.

1905. (a) Consider the graphs $y = x^2$, $y = x^4$, etc.
(b) Consider $y = x^3$, $y = x^5$, etc.

1906. Integrate the DE three times. Each time you do, introduce a new constant of integration. You will then need to integrate that constant in the next stage.

1907. (a) Use geometry to find the angle between the sling and the horizontal. Then use NII.
(b) Consider the size of the ball-bearings.

1908. Substitute 75° into the identity, and then use the fact that $\cot 75° = \frac{1}{\tan 75°}$.

1909. Consider the boundary cases individually, finding the greatest and least value for the lowest value.

1910. The denominator of the LHS is a difference of two squares.

1911. In each case, the rightwards implication obviously holds. So, work out whether there are solutions to the right-hand equations which do not have $x = y$.

1912. Factorise top and bottom. If you can't spot the factors directly, do a bit of reverse engineering: solve e.g. $3x^2 + 5x + 2 = 0$ and use the factor theorem.

1913. Use the standard (a, b, c) lengths, with c as the hypotenuse. Calculate the overall area as c^2, and then as the sum of the four shaded triangles and the central square.

1914. The statement is true. Use the standard formula for conditional probability.

1915. Consider three concurrent lines.

1916. Find the relevant stationary point using the first derivative, and then show algebraically that the second derivative changes sign at this point.

1917. Consider the fact that an arithmetic sequence could also be called a linear sequence.

1918. (a) Substitute the value $t = 0$.
(b) Equate the positions and solve for t. Then sub this back into either one of the displacement formulae.
(c) Consider the behaviour as $t \to \infty$.

1919. You need to solve fully. It isn't enough to check the discriminants of each factor here, because the right-hand factor is a biquadratic.

1920. Use the chain rule to differentiate. Then, having found $\frac{dy}{dx}$ and y at $x = \frac{\pi}{4}$, substitute the values into $y - y_1 = m(x - x_1)$.

1921. Consider the interior of the circle as a possibility space. Sketch it, and sketch the region satisfying the inequality.

1922. There is a sign change at a single and a triple root, but not at a double root. Both double roots and triple roots are repeated roots, where the x axis is a tangent.

1923. Complete the square on the general quadratic, and then read off the relevant transformations from the resulting expression.

1924. In both (a) and (b), you can use either a symmetry argument, or a compound angle formula.

1925. (a) Find the vertex of the parabola $y = \mathrm{g}(x)$.
(b) Consider that $\mathrm{f}(x)$ is increasing everywhere.

1926. Apply the differential operator $\frac{d}{dx}$ to both terms in the bracket. Then rearrange.

1927. Consider the possibility space as the set of possible locations of the red counters. Hence, use ${}^n\mathrm{C}_r$ for both parts.

1928. These are two circles. Complete the square and show that the unit circle lies entirely within the other circle.

1929. Differentiate and find the turning points, or else factorise and consider the multiplicity of the roots.

1930. Consider the values for which the denominator is zero, recognising that the index $2k$ is even.

1931. This is a quadratic in $\operatorname{cosec} x$. Rearrange to the form LHS $= 0$ and factorise. There are three roots in $[-\pi, \pi]$.

1932. It is true that the IQR cannot exceed the range. So, find a counterexample in which the IQR is equal to the range.

1933. Consider each graph as a cubic centred on the point (a, b). You might find it easier to visualise having multiplied up by the denominators.

1934. Solve $x^3 + x + 2 = 0$ first, then sketch the graph, then set up the relevant definite integral.

1935. Sketch the hexagon, and note that opposite sides have the same vector.

1936. Since inverse functions, in order to be invertible, must be one-to-one, the domain of each function must be the same set as the codomain of the other.

1937. A fixed point satisfies $x = \mathrm{f}(x)$. Set this equation up, and solve it.

1938. Provide a counterexample. This means looking for a specific function f and two specific x values a and b, for which $y = \mathrm{f}(x)$ has a change of sign between $x = a$ and $x = b$, but does not cross the x axis between them.

1939. Use the formula $\mathbb{P}(X \mid Y) = \dfrac{\mathbb{P}(X \cap Y)}{\mathbb{P}(Y)}$.

1940. The Newton-Raphson method is sometimes called *tangent-sliding*. It uses a linear approximation to the function f in $\mathrm{f}(x) = 0$, i.e. a tangent to $y = \mathrm{f}(x)$, and solves to find where the tangent crosses the x axis. The process is then repeated. Possible pitfalls are ① if the new approximation x_1 is outside the domain of the function, or ② if the gradient of the tangent is small/zero.

1941. The answer to this question is a useful thing to know as a quoted fact, especially when solving equations involving modulus functions. To work it out from scratch, find the implication between each of these statements and a third statement: $x = \pm y$.

1942. Neither is true. The two functions $\mathrm{f}(x) = |x|$ and $\mathrm{g}(x) = -x$ form a counterexample to each of the statements. Explain why.

1943. A stretch in the x direction is a replacement of x with kx. Hence, write 3^x as 2^{kx}.

1944. With inequalities more complicated than linear, the technique is to solve the associated boundary equation, to find the boundaries of the region you are looking for, and then consider the behaviour (using graphs, generally) of the relevant functions. In this case, since the result is *always* true, you would expect the associated quadratic equation to have no real roots.

1945. Before answering this question, you might consider the simpler example in which $k = 0$. The results found there will generalise, since replacing x with $(x - k)$ is a translation in the x direction.

1946. The difference between the two expression is that, in the LH expression, the value of t is set to be a. In the RH expression, however, the value of t is never a, it only approaches arbitrarily close to a. The common factor $(t - a)$, which can be found on the top and bottom, can therefore be cancelled in the limit expression.

1947. It doesn't matter where the first face is. Hence, the only consideration is the number of successful and total outcomes for the second choice of face.

1948. Every fifth integer is divisible by 5. So, the first 125 will contain 25 numbers which are divisible by 5, hence 100 which are not divisible by 5.

Find the sum of the first 125 natural numbers, and also the sum of the numbers which *are* divisible by 5, and subtract them. Both are sums of APs.

1949. First, multiply up by both denominators, in order to set up a polynomial equation. Then, use the factor theorem: since $x = 3$ is a root, you know that $(x - 3)$ is a factor. Taking this factor out leaves you with a quadratic.

1950. Positive or negative *first* derivative (i.e. gradient) means increasing or decreasing.

Positive or negative *second* derivative (curvature) means convex or concave.

1951. In each case, use the index law $\left(a^b\right)^c \equiv a^{bc}$. In (b) and (c), also use $a^{b+c} \equiv a^b a^c$.

1952. The easiest way to prove this is by dividing top and bottom by x, before then letting x tend towards infinity.

1953. Consider the graph as a transformation of $y = |x|$. There is a stretch, scale factor 2, in the y direction, then a *reflection* (ignoring this is the error) in the y axis, then a translation by 3 units in the positive y direction.

1954. Vectors $x_1\mathbf{i} + y_1\mathbf{j}$ and $x_2\mathbf{i} + y_2\mathbf{j}$ are perpendicular if and only if

$$\frac{y_1}{x_1} = -\frac{x_2}{y_2}.$$

Use this to set up an equation, and solve for x.

1955. The lengths of the perpendicular sides are given by $x = 3 + t$ and $y = 4 + t$. Use these to find an expression for the hypotenuse z in terms of t. Differentiate it with respect to time.

1956. (a) As always, a sketch will be helpful. Consider the symmetry of the distribution.

(b) Use $y = 4$, below which the probability is 0.8. Using a calculator, evaluate $\Phi^{-1}(0.8)$, which will tell you how many standard deviations away from the mean $y = 4$ is. Set up and solve the equation $\sigma \times \Phi^{-1}(0.8) = 2$.

1957. (a) A function is invertible iff it is one-to-one, with values in the domain and codomain matched up. Consider whether multiple inputs map to the same output.

(b) A sketch will help.

(c) For an invertible function, the codomain must be the same as the range.

1958. (a) If the inequality has the solution set given, then the boundary equation has solution set $\{4, 5\}$. But there are infinitely many quadratic equations which have this solution set.

(b) Consider whether $y = ax^2 + bx + c$ is a positive or negative parabola.

1959. This inequality has a pair of boundary equations:

$$(x-2)^2 + (y-3)^2 = 4,$$
$$(x-2)^2 + (y-3)^2 = 9.$$

Sketch these first.

1960. A local maximum is a point with first derivative $\frac{dy}{dx} = 0$ and second derivative $\frac{d^2y}{dx^2} < 0$. Factorise to solve the resulting equation.

1961. These questions don't require use of the normal distribution machinery, only consideration of its symmetry. The key fact is that the two outcomes $Z > 0$ and $Z < 0$ are equally likely, making them equivalent to heads/tails on a coin.

1962. The length of the cylinder isn't relevant, since the proportion of volume occupied will be the same whatever the length. So, you can work with a semicircular cross-section. Find the area of the segment (via sector minus triangle) as a fraction of the area of the semicircle.

1963. (a) Use -1 in place of n and $4x$ in place of x.

(b) Substitute $4x$ into the validity condition.

1964. Rearrange the equation to equal zero before taking out a factor of $(3x - 2)$.

1965. Since $\sin^2\theta + \cos^2\theta \equiv 1$ is an identity, it can be used without affecting any implications. Convert $\sin^2\theta$ before considering the implication.

1966. Convert the angular speed in radians per second into arc length per second, using $l = r\theta$.

1967. (a) Colour one triangle, and the rest follow.

(b) Quote a standard result.

1968. The associated equation has no real roots. That doesn't mean, however, that the solution set for the inequality is empty. A sketch of a parabola will show you what's going on.

1969. The two graphs are the same, except the x's and y's have been swapped. So, they are reflections of each other in the line $y = x$. Hence, wherever they intersect with $y = x$, they intersect with each other. Use this to look for points of tangency.

1970. Use the same technique as is usually employed for turning recurring decimals into fractions. Multiply the equation by 10^n, then subtract the equations.

1971. The equation associated with each inequality is a pair of straight lines. Sketch these first.

1972. The instantaneous velocity is given by $v = 3t^2 + 2$. The average velocity is found by integrating v over $[0, 4]$, before dividing by the total time.

1973. Factorise to $(1 + x)^5(1 + 2x)^5$, then expand each factor up to terms in x^2. You don't then need to multiply this out: pick the three relevant terms.

1974. This is a pair of linear simultaneous equations in $\sin x$ and $\cos x$. Elimination is probably easier than substitution.

1975. The obvious features of the normal distribution are that it is unimodal and symmetrical. Neither feature is likely to appear in the population being modelled.

1976. It's much easier to find the correct route through the algebra by starting with the more complicated equation. However, in simplifying, care is required when considering the direction of the implications.

1977. Set up a new function f as the difference between the y values of the two curves. Calculate a definite integral to find I, the true value of the area, and use the trapezium rule as suggested to find T, an approximation to I. The percentage error is given by $(T - I)/I$.

1978. Perform the two integrals, and combine the $+c_1$ and $+c_2$ into a single constant.

1979. Consider a quadratic of the form $y = \mathrm{f}(x)$ and a cubic of the form $x = \mathrm{f}(y)$. As ever, a sketch will be helpful.

1980. (a) This function is quadratic in 2^x. Hence, its range is $y \geq k$, where k is the y coordinate of the vertex. This can be found by setting $z = 2^x$ and then completing the square.
(b) Factorise into the form $(4z - 1)(...)$.

1981. You can think of this in the language of implicit differentiation. The differential operator $\frac{d}{dx}$ stands for "differentiate (either implicitly or explicitly) with respect to x". Enact the differentiation of the LHS using the chain rule, with the square root function as the outside function.

1982. (a) Differentiate using the chain and product or quotient rule, and evaluate.
(b) Consider the fact that the gradient in (a) is small and *negative*: locate the relevant point on the graph and run the iteration graphically.

1983. This is the third Pythagorean trig identity. It can be derived by dividing the first Pythagorean trig identity by $\sin^2\theta$.

1984. (a) Differentiate $z = x^2 + 4$ with respect to x, and rearrange to the required form.
(b) Enact the substitution, using part (a).

1985. The notation $\mathrm{f}^3(x)$ means $\mathrm{f}(\mathrm{f}(\mathrm{f}(x)))$. So, first find and simplify $\mathrm{f}^2(x)$, by setting the input of $\mathrm{f}(*)$ as $* = \mathrm{f}(x)$. Then repeat the process.

1986. By symmetry, this is an equilateral triangle. Hence, to find its area, you only need to find the length of AB. This is the diagonal of a square.

1987. In each case, list the successful outcomes. Then find their individual probabilities and add them.

1988. You are being asked to verify that $\mathrm{g}(x) = \sqrt{x} + 1$ is a solution of the differential equation given. So, calculate g', and substitute both g and g' into the left-hand side, as an expression. Show that this expression simplifies to 1.

1989. Multiply out the difference of two squares, and use the fact that $|x|^2 \equiv x^2$.

1990. Don't be put off by the complicated-looking sum. The first term of the series is simply what follows the Σ, with $k = 0$ substituted. Evaluate this using a calculator.

1991. The average value of a function f is given by the continuous sum, i.e. definite integral, of $\mathrm{f}(x)$ over the domain, divided by the width of the domain.

1992. Rewrite the equation in terms of $\mathbb{P}(X = 0)$ and $\mathbb{P}(X = 1)$, using the usual formula for conditional probability. Express these in terms of p, and solve.

1993. (a) Use $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$.
(b) Set $\frac{dy}{dx} = 1$ and solve.

1994. Evaluate the second derivative at $x = 3$.

1995. Solve to find the vertex of the parabola $y = a_n - b_n$.

1996. This is true. Just differentiate the first statement.

1997. Explain why B and E will cancel out, and why C will exert no moment. Then it comes down to which of D and A exerts the larger moment.

1998. Let $x = \sqrt{y}$. Solve for x by multiplying up by the denominators of the fractions.

1999. This is incorrect.

2000. Calculate the derivative of $\cot x$ (or look it up) and substitute. You'll need a Pythagorean trig identity.

——— End of Volume II ———